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On the Modulus of Torsional Rigidity of Quartz Fibres and Its Temperature Coefficient

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XI. *On the Modulus of Torsional Rigidity of Quartz Fibres and its Temperature Coefficient.*

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THE author recently had the honour of reading to the Society the results of some experiments in which a new method of timing by means of "coincidences"—a method devised by Professor POYNTING—was applied to an investigation of the effects of changes of temperature on the modulus of torsional rigidity of metal wires. It seemed desirable to repeat similar experiments with quartz fibres, seeing that now they are almost universally employed as suspensions in torsion instruments where accuracy is required.

The modulus of torsional rigidity of quartz fibres was first investigated by C. V. BOYS ('Journal Soc. of Arts,' vol. 37, p. 827, 1889), who found considerable variations between different specimens. The mean value of the modulus obtained by him was $n = 2{\cdot}38 \times 10^{11}$ dynes per sq. centim.

An independent study of the subject was made a little later by THRELFALL ('Phil. Mag.,' V., vol. 30, p. 99, 1890), who also investigated the effect on the rigidity of heating a fibre to 100° C. It was found that the rigidity increased with the temperature, and, assuming that the change of rigidity with temperature obeyed a linear law, the value obtained for the temperature coefficient of the modulus was $+{\cdot}000128$. The mean value of the modulus of rigidity was found to be $2{\cdot}88 \times 10^{11}$ dynes per sq. centim., but the different determinations were not in very good agreement.

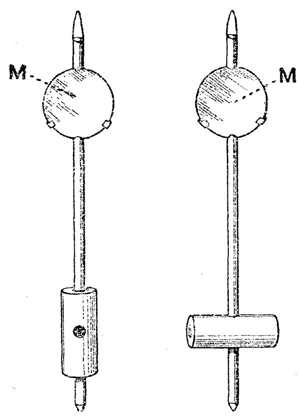
BARNETT ('Phys. Rev.,' vol. VI., p. 114, 1898) has also investigated both the modulus of rigidity and its temperature variation. From observations on six fibres he obtained a mean value $n = 1{\cdot}46 \times 10^{11}$ dynes per sq. centim., or about half the value obtained by BOYS and THRELFALL. The relation between the rigidity and the temperature, for temperatures ranging from 20° C. to 100° C., was shown to be linear by timing the torsional oscillations of a fibre at several temperatures between these limits. The mean value of the temperature coefficient of the modulus was +·000115, but the agreement between the values obtained from different fibres was not very good.

The object in repeating these experiments was to apply, where possible, more accurate methods for obtaining the data required in order to calculate the rigidity modulus. In the experiments about to be described the points to which particular attention was directed were the timing of the torsional oscillations of the fibres and the determination of their radii. The experiments were divided into three parts :—

(1) The determination of the absolute value of the torsional modulus.
(2) The variation of the modulus over a range of temperatures from 15° C. to 100° C.
(3) The variation of the modulus between 20° C. and 1000° C.

PART I.—THE DETERMINATION OF THE MODULUS OF TORSIONAL RIGIDITY.

The fibres experimented on were prepared from different crystals of quartz by means of the bow-and-arrow process originally used by BOYS.* Great care was taken in their manufacture in order to obtain them free from air bubbles and of circular cross-section. A length of from 10 to 15 centims. of a selected fibre was mounted as the suspension of a small vibrator and enclosed in a double-walled jacket shown diagrammatically in fig. 2. The vibrator, the total mass of which was 2·7 grammes, is shown on a larger scale in fig. 1. It consists of two parts; a brass rod, 1·5 millims. in diameter and 8 centims. in length, and a small brass cylinder about 11 millims. long by 5·5 millims. in diameter. This cylinder has two holes drilled through it, one along its axis, and the other at right angles to the axis at the middle of its length. These holes are of the same diameter as the rod, and thus the cylinder can be fixed on the rod in the two positions shown in fig. 1. In this way it is possible to obtain, experimentally, the moment of inertia of the vibrator without adding to its mass. This precaution seemed necessary in view of the statement by BOYS† that the rigidity of the fibres varies with the tension to which they are subjected. A small plane mirror M fixed upon the rod serves as a means of observing the torsional oscillations. The upper end of the vibrator rod is pointed, and one half filed away in order that the fibres may be attached along the axis. The upper end of the fibre is attached to a brass pin which can be clamped in



First position. Second position.

Fig. 1. The vibrator.

* C. V. BOYS, 'Phil. Mag.,' V., vol. 23, p. 489, 1887.

† C. V. BOYS, 'Phil. Trans.,' A, vol. 186, p. 65, 1895.

the end of the steel rod E (fig. 2). This passes through the torsion head D, and its length is sufficient to allow of its end being lowered so as to be well outside the jacket, for the purposes of fixing the fibre in position and of measuring its length. The fibres were attached to the brass pin and to the vibrator sometimes by shellac, sometimes by silvering, electroplating with copper and soldering, and sometimes by means of a tooth-stopping cement, suggested to me as a suitable material by Mr. C. V. Boys, and supplied by Messrs. C. Ash, Limited, of Golden Square, W. This last method has the advantage that it does not involve heating the fibre, and thereby possibly altering its rigidity. It was found to give a suspension free from zero changes, and in every respect as good as shellac. I had hoped that it would withstand the effects of heat, but the cement always cracked away from the metal at about 50° C.

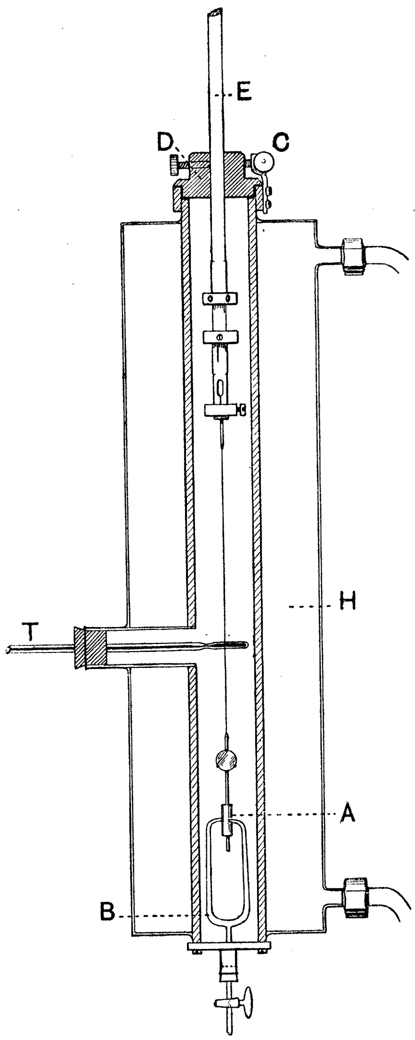


Fig. 2.

The jacket H consists of two concentric tubes. The outer one is of brass, 3 inches in diameter, and is covered externally with layers of cotton wool, around which a piece of sheet asbestos is wrapped to protect it from fire. The inner tube is of copper, $\frac{1}{8}$ inch in thickness and 1 inch in internal diameter. This has two side tubes, each $\frac{1}{2}$ inch in diameter. One of these is at the middle of the jacket, and through this the thermometer T, used to indicate the temperature of the jacket, projects. The second side tube, which is not shown in the figure, is at right angles to the first and nearer to the lower end of the jacket. The outer end of this tube is covered with a piece of optically worked glass, and through it the mirror on the vibrator can be seen.

In order to set the vibrator in oscillation without swinging it like a pendulum bob, the following device, already described in a previous paper,* was adopted. Two fine glass jets were arranged close to the vibrator and in such positions that when a stream of air was blown through them it would produce a couple on the vibrator and so rotate it. These jets (B, fig. 2) are joined at the bottom of the jacket to a glass tube, which passes out through a cork and is connected to 3 or 4 yards of fine "composition" tubing passing along the wall of the laboratory and ending on a small table beside the observing telescope. Here it is joined to a piece of indiarubber

* F. Horton, 'Phil. Trans.,' A, vol. 204, p. 1, 1904.

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tubing, the other end of which is closed, and by gently squeezing this any desired amplitude of oscillation can be communicated to the vibrator.

The clamping rod E had been used in the experiments with metal wires, and, in order to prevent leakage of heat along it, a piece of marble had been inserted between the clamp and the rest of the rod. The rod can be fixed at any desired level in the brass head D, which works smoothly in a collar and is capable of rotation about a vertical axis by means of the screw C, thus giving a fine adjustment to the azimuth in which the vibrator hangs.

In determining the absolute value of the rigidity modulus, a rapid stream of water was allowed to circulate through the jacket. It would then take about an hour for the temperature, as indicated on the thermometer, to become constant.

Professor POYNTING'S modification of the method of coincidences, which was used in timing the torsional oscillations, has been described in detail in the paper before referred to. A description of the apparatus used to obtain the second flashes is also contained in the same paper. It will be sufficient to mention here that the accuracy in timing obtained in the present experiments was about 1 in 100,000.

The thermometer T was graduated to ·1° C. and had been standardised for use horizontally at the National Physical Laboratory. The part of the stem protruding from the jacket was surrounded by a glass tube, through which a stream of water at a known temperature was kept flowing, and thus the correction for the exposed part of the mercury thread was obtained. It was only necessary to apply this correction in the experiments at higher temperatures.

The Order of Experiment.

A fibre of the required size having been mounted in the manner already described, its length was first measured by means of a cathetometer. For this purpose the rod E was left in the position to which it had been lowered in order to clamp the brass pin to which the fibre was affixed. The vibrator and fibre were surrounded by a glass tube to keep off air currents, and the ends of the fibre were viewed in turn by the cathetometer telescope. Owing to the very small coefficient of expansion of quartz, it was unnecessary to take precautions to maintain the fibre at an accurately known temperature whilst its length was being measured. The measurement of the length was repeated before the fibre was taken down, but it was always found to be constant, showing that the fibre had not drawn away from the cement, or other material, used to fix it in position.

After the length of the fibre had been measured, the clamping rod was carefully raised until the vibrator was at the proper height inside the jacket. This was an operation requiring much care and patience, for any irregular motion set the vibrator in violent agitation, and with the finer fibres always ended in their breaking. Vibrations were somewhat avoided by allowing the vibrator to rest against the side of the glass tube as it was being drawn up, but this could not well be managed after the fibre had disappeared inside the jacket. The torsion head was next arranged so that the mirror on the vibrator reflected an image of the slit, used in the timing arrangements, into the observing telescope. The copper bottom was then screwed on to the jacket, a lead washer being compressed between, so as to make an air-tight joint. The glass jets for setting the vibrator in oscillation were next arranged in position, and any final adjustment required in the position of the mirror was made by means of the fine screw C which rotates the torsion head. A rapid stream of water was then sent through the jacket, and the apparatus was allowed to rest until a constant temperature had been attained.

In determining the period of vibration the mean of some 10 or 15 observations was taken. The bottom of the jacket was then removed, the vibrator lowered outside, and the small cylinder slipped off the rod and replaced with its axis perpendicular to its first position, as shown in fig. 1. It was then raised again into the jacket, adjusted into position, and allowed to rest until a constant temperature had been established, when the torsional oscillations were timed as before. The vibrator was then again lowered and the cylindrical part replaced in its first position, after which the period was again observed. The mean of this and the first observation was taken as the period corresponding to the moment of inertia of the vibrator with the cylinder in the first position, and was used with the other observation to give the modulus of rigidity.

Corrections to the Observed Periods of Vibration.

The value of the rigidity modulus to be determined was that corresponding to 15° C. It was therefore necessary to correct the observed periods to the values they would have had if the temperature during the observations had been 15° C. The periods were therefore first corrected for clock rate and then multiplied by the factor

$$1-(\beta-\tfrac{3}{2}\alpha-\tfrac{1}{2}\gamma)(\theta-15),$$

in which β is the coefficient of linear expansion of the material of the vibrator, α is the coefficient of linear expansion of quartz, γ is the temperature coefficient of the modulus of rigidity of the fibre.

Of these, β was determined experimentally by measuring the coefficient of expansion of the rod of brass from which the vibrator was afterwards made. This was done with the measuring bench in the Physical Laboratory of the University of Birmingham, for the use of which I am indebted to Professor POYNTING. The instrument and method of experiment are described in the former paper. The mean value found for the coefficient of expansion was ·00001937.

The correction on account of the expansion of the fibre is very small, owing to the smallness of the coefficient of expansion of quartz. It was therefore not thought

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necessary to make a special determination of this, especially seeing that the subject has been recently investigated by several observers.* The value used for α in the above factor is that given by HOLBORN and HENNING, viz., ·00000054. The method by which γ was determined forms Part II. of this paper. In some cases the temperature coefficient was actually found for the fibre in question, in others a mean value was taken.

The logarithmic decrements of the torsional oscillations of the fibres were found at the same time as the periods of vibration. The correction to the observed period on account of the damping of the vibrations was applied when necessary.

The Measurement of the Radii of the Fibres.

When the observations of the torsional vibrations were completed, the fibre was removed from the jacket, and its mean radius was determined. For this purpose the apparatus shown in fig. 3 was used. The method consists in measuring the circumference of the fibre by rolling it between two fine glass tubes and counting the number of revolutions it makes in travelling a distance of 5 millims.

The glass tubes A and B are capillary tubes of uniform external diameter. B is fixed by means of wax on to a small adjustable table D, shown on a larger scale in fig. 4. The height of this can be carefully adjusted by means of the milled head E, and the table can be set horizontally by levelling screws. A was drawn down from a wider tube which is firmly attached to the brass sliding table F. This moves along two slides, one of which is V-grooved and the other plane, after the manner usually adopted in travelling microscopes. The table F can be pushed forward by turning the milled head H, which is attached to a steel screw with a $\frac{1}{2}$-millim. pitch thread, so that a slow motion of the table can easily be made. The distance through which the table moves is measured by means of a fine Zeiss scale on glass, which lies above a circular aperture in the centre of the table and is illuminated from below. The scale is viewed from above by the microscope M, fitted with a rocking-plate micrometer,† by means of which measurements to ·00002 centim. can be made.

In order that the fibre shall roll, it has to be perfectly free from dirt or grease. The tubes A and B were generally renewed for each fibre; the glass, when freshly drawn out and afterwards protected from dust, being found to work better than when cleaned in the ordinary way.

A length of about 1 centim. of the fibre is experimented on at a time. This is prepared in the following manner:—The end of the fibre to be measured is first well cleaned in chromic acid and afterwards washed in water. It is then placed transversely

* SCHEEL, 'Ber. d. Deut. Phys. Ges.,' 5, 1903; HOLBORN and HENNING, 'Ann. d. Phys.,' 10, 2, 1903; P. CHAPPIUS, 'Verh. Naturf. Ges. Basel,' 16, 1903.

† For description see POYNTING, 'Phil. Trans.,' A, vol. 182, p. 589, 1891, or 'The Mean Density of the Earth,' p. 95.

between the two capillary tubes and cut through at about 1 centim. from the end, so as to leave this 1 centim. of the fibre lying transversely between the glass tubes.

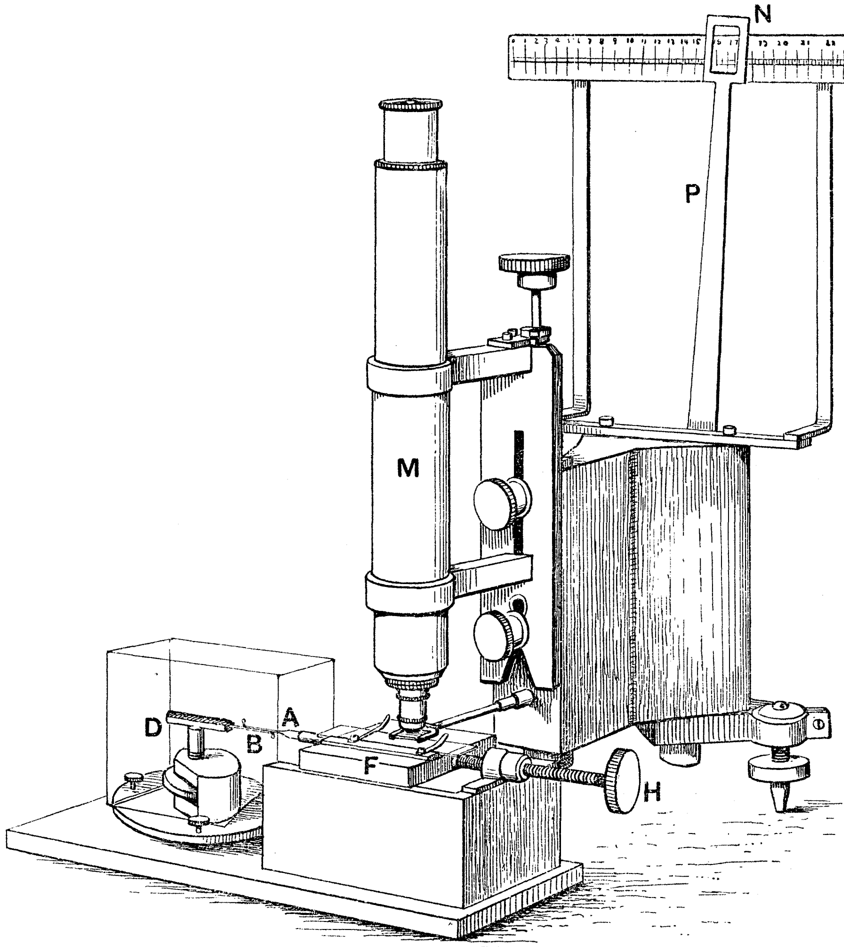

Fig. 3.

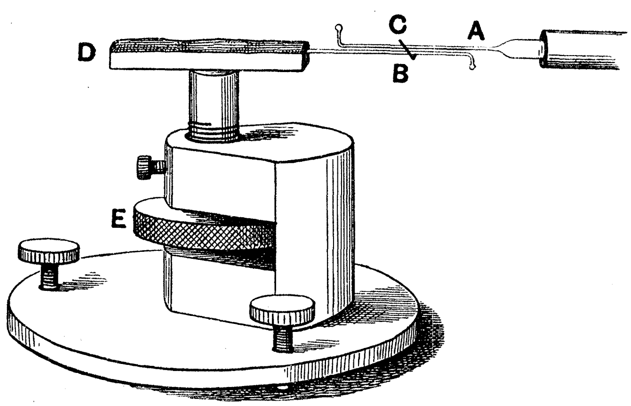

Fig. 4.

About 1 millim. of the end nearest to the observer of this piece of the fibre is then turned up at right angles to its length by means of a very small gas flame. This

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turned-up end serves as an index for counting the revolutions of the fibre. It is viewed by means of a horizontal microscope not shown in the figure. The table D has next to be adjusted in height and in horizontality until the fibre rolls quite freely on sliding the table F along its guides. When once these adjustments have been made they serve for the whole fibre—in fact, until it becomes necessary to renew the glass capillary tubes on account of their becoming dirty. The small table D is covered with a glass shade which extends over the tubes A and B and protects them from dust (see fig. 3).

The Zeiss scale is 1 centim. in length. It is divided into millimetres, the last of which is subdivided into tenths. The scale was standardised by comparison with the standard metre of the Laboratory, several centimetres of the latter being measured in terms of the Zeiss scale at a temperature of 15° C. The agreement, after the necessary temperature corrections had been made, was within the limits of experimental error, and the scale was therefore taken to be correct at 15° C. The temperature of the scale during the measurements of the radii of the fibres was noted, but the temperature corrections to the length of the scale were always too small to be observed.

Before taking an observation it is first necessary to see that the scale is fixed on the sliding table so as to be parallel to the direction of motion of the latter. In order to enable this to be easily done, a strip of metal was screwed on to the table so as to be accurately parallel to the slides, and the scale was held against this by means of two brass springs.

The whole of the apparatus, except the microscope M, is fixed on one heavy brass base, and thus motion between the fixed table D and the moving one F, other than that measured on the scale, is avoided. The apparatus can be levelled so that the scale is horizontal. This is tested by the definition of the image in the microscope remaining perfect as the scale is moved along underneath. The microscope M is rigidly attached to a heavy cast-iron support, which rests on levelling screws, so that the axis of the microscope can be adjusted into the vertical. The apparatus, when in use, stands on a stone slab which cannot be moved by accidental blows.

In taking an observation, the moving table is drawn back until the zero on the scale has passed the cross-wire of the first or vertical microscope. The head H is then turned until the zero is seen approaching the cross-wire, and then the fibre is viewed through the second or horizontal microscope. The head H is further turned until the index appears vertical, *i.e.*, parallel to a vertical cross-wire in the second microscope. This is a position which can be obtained to within a very small fraction of a revolution. The image of the zero of the scale is then adjusted on to the cross-wire of the microscope M by means of the rocking plate, and the position of the latter is noted by reading the position of the arm P on the scale N. The head H is then slowly and uniformly rotated and the revolutions of the fibre counted. This is continued until such a number of exact revolutions have been made that the last ·1 millim. of the scale comes under the cross-wire. The position of the cross-wire in this last ·1 millim. is then found by measuring the amounts through which the rocking plate has to be tilted, in order to bring the bounding lines, in turn, on to the cross-wire. In this way the number of revolutions made by the fibre while the upper glass tube travels a known distance over the lower one is obtained. The fibre itself evidently travels through half this distance, and so, dividing the measured distance by twice the number of rotations counted, the circumference of the fibre is obtained, and hence its radius.

Each piece of the fibre was measured in three places, and there were usually 10 or 12 pieces. The mean of all the measurements was taken as the radius of the fibre. The agreement between repeated measurements of the radius at the same part of a fibre was always very close. A single reading could be trusted to ·01 per cent. The diameter of most of the fibres was found to vary along their length, generally in such a manner as to indicate that they were slightly conical in shape, but sometimes in an irregular way. The difference between the largest and smallest diameters was generally less than 1 per cent. of the whole, but it was sometimes more. The greatest difference found during these experiments was 4 per cent. in the case of a fibre 14 centims. long and of mean diameter ·0010256 centim., as found by measurements at 40 places approximately equally spaced along its length. In order to illustrate the accuracy of this method of measuring small diameters, I give below the results of four consecutive observations on one of the first fibres measured. These are repeated measurements of the same part of a fibre, the mean diameter of which was ·0012483 centim.

In 127 revolutions of fibre the scale travelled (*a*) ·99609 centim.; (*b*) ·99609 centim.; (*c*) ·99608 centim.; (*d*) ·99609 centim.

When differences between two successive readings were obtained, it is probable that they were due to a slightly different part of the fibre being traversed on each occasion. This easily happens unless great care is taken in drawing back the sliding table after an observation has been taken.

As an example of the manner in which the diameter of a fibre varies at different points, I give below the values of the diameter of the first fibre experimented on, as measured at 20 places approximately equally spaced along its length.

Diameter of Fibre in Centimetres.

·0021090,	·0021184,	·0021160,	·0021145,
·0021071,	·0021172,	·0021175,	·0021150,
·0021103,	·0021191,	·0021200,	·0021228,
·0021080,	·0021132,	·0021219,	·0021182,
·0021130,	·0021149,	·0021256,	·0021181,

Mean diameter = ·0021160 centim.

The Results of the Experiments.

In order to calculate the modulus of rigidity it is necessary to know the moment of inertia of the movable cylinder of the vibrator when in the two positions shown in fig. 1. The mass and dimensions of the cylinder were therefore accurately determined. They were

Mass of cylinder =	2·0987 grammes,		
Mean diameter of cylinder =	5·5841 millims.	at	13·5° C.,
Diameter of longitudinal hole =	1·4490 ,,	,,	13·5° C.,
Length of cylinder =	11·0202 ,,	,,	13·5° C.,
Diameter of transverse hole =	1·4956 ,,	,,	13·5° C.,

from which moment of inertia of the cylinder with its axis vertical (fig. 1, first position) = ·0878115 gramme cm.^{2} at 13·5° C., and moment of inertia about the axis of rotation when in the second position (fig. 1) = ·263747 gramme cm.^{2} at 13·5° C.

The coefficient of expansion of the cylinder had been determined and was ·00001937, from which the moments of inertia of the cylinder in the two positions at the standard temperature 15° C. are

$$I' = \cdot 263762 \text{ gramme cm.}^2, \quad I = \cdot 087817 \text{ gramme cm.}^2$$

The periods obtained with the first fibre experimented on were

Cylinder on vibrator in first position,	$T_1 =$ 8·47140	at	13·11° C.,
,, ,, ,, ,, second ,,	$T' =$ 14·34857	,,	12·99° C.,
,, ,, ,, ,, first ,,	$T_2 =$ 8·46916	,,	13·15° C.,

Mean of the two periods in first position, T = 8·47028 at 13·13° C.

These periods, T and T′, have now to be corrected to the values they would have if the temperature had been 15° C.; correcting, therefore, on account of the alteration of rigidity of the fibre and of the dimensions of the fibre and vibrator, we get the corrected periods

T = 8·46967 secs. and T' = 14·34745 secs.,
Length of the fibre, l = 11·268 centims.,
Mean radius, r = ·001058 centim.

The formula for the modulus of rigidity is

$$n = \frac{8\pi l\,(I'-I)}{r^4\,(T'^2-T^2)},$$

which gives for the modulus of rigidity of this fibre at 15° C. the value

$$2\cdot965 \times 10^{11} \text{ dynes per sq. centim.}$$

In all, the modulus of rigidity of six fibres was determined. These were prepared from different specimens of quartz, in order to see to what extent the modulus could be taken as being a constant quantity. The first fibre experimented on—that to which the above numbers refer—was taken from the laboratory stock of fibres, and was probably not so carefully prepared as the others. The modulus for this fibre is, however, in quite good agreement with the other values obtained. These are given in the following table :—

Fibre.	Mean radius in centimetres.	Length in centimetres.	Modulus of rigidity at 15° C.
1	·0010580	11·268	$2 \cdot 965 \times 10^{11}$ dynes per sq. centim.
2	·0005969	14·8645	$2 \cdot 788 \times 10^{11}$ „ „
3	·0005711	14·505	$2 \cdot 986 \times 10^{11}$ „ „
4	·0011239	14·049	$3 \cdot 018 \times 10^{11}$ „ „
5	·0007078	14·338	$2 \cdot 992 \times 10^{11}$ „ „
6	·0005128	13·6055	$3 \cdot 042 \times 10^{11}$ „ „

It should be mentioned that fibres 4, 5, and 6 in the above table had been heated several times over a range of temperature from 15° C. to 100° C. in determining the temperature coefficient of the modulus, as described in Part II. of this paper. The effect of this heating is slightly to increase the value of the modulus at the ordinary temperature of the laboratory, but this increase is much too small to account for the fact that the rigidity of these fibres is greater than the other values given in the table.

The diameter of fibre 2 was found to vary in an irregular manner at different points along its length. The value given as the mean radius of this fibre is therefore not so reliable as in the other cases. This inaccurate knowledge of the radius probably accounts for the value of the modulus of rigidity given by fibre 2 being considerably lower than the other values obtained. The mean of these other values, viz.,

$$n = 3 \cdot 001 \times 10^{11} \text{ dynes per sq. centim.,}$$

may be taken as the modulus of rigidity of quartz fibres at 15° C.

From the numbers in the table given above it is seen that there is no sort of connection between the radius of the fibre and its modulus of rigidity. It follows that, since the same vibrator was used with each fibre, the modulus of rigidity is independent of the longitudinal stress to which the fibre is subjected so long as this is kept within the limits used in these experiments. The stress in each case is given below.

Radius in centimetres.	Longitudinal stress in 10^8 dynes per sq. centim.	Modulus of rigidity at 15° C. in 10^{11} dynes per sq. centim.
·0011239	6·67	3·018
·0010580	7·53	2·965
·0007078	16·84	2·992
·0005711	25·85	2·986
·0005128	32·07	3·042

In his gravitational experiments, C. V. Boys* found that the rigidity of quartz fibres diminished under increased longitudinal stress. The fibres used in his experiments, however, were subjected to greater longitudinal stress than those used in the present research, being loaded, of necessity, as nearly to the breaking-point as was safe. It is probable, therefore, that the decrease of rigidity with increasing tension obtained by Mr. Boys only occurs when the fibre is loaded nearly to its breaking-point.

Part II.—The Variation of the Modulus of Rigidity between 15° C. and 100° C.

In order to make an accurate study of the effects of changes of temperature on the modulus of rigidity of the fibres, observations of the periods of torsional oscillation were made at several temperatures between that of the room and 100° C. These temperatures were obtained by passing the vapours of various liquids, boiling under atmospheric pressure, through the jacket H of the apparatus shown in fig. 2. The liquids used were ether (35° C.), acetone (55° C.), methylated spirit (75° C.), and water (100° C.). In order that the composition of the liquid, and therefore its boiling-point, might not vary, a reflux arrangement was used, the vapour being driven from a large copper boiler through the heating jacket, after which it is condensed by a condenser, the condensed liquid running back again into the boiler.

The fibres used in these experiments were fixed in position by the process of silvering, coppering, and soldering, the temperatures used preventing the more simple methods of fixing being applied.

In a previous research on the rigidity of metal wires at various temperatures it was found that the modulus of rigidity, in almost every case examined, gradually increased as time went on. This increase was generally very small, and was greater at higher temperatures than at the ordinary temperature of the laboratory. In order to see if such an effect was given by quartz, observations were made at intervals, while the fibres were kept continuously at a constant temperature. In the course of

* C. V. Boys, 'Phil. Trans.,' A, vol. 186, p. 61, 1895.

the observations for the absolute value of the modulus it had already been observed that the period of torsional vibration diminished slightly as time went on, thus indicating an increase in the rigidity of the fibre. This was also shown in some of the observations at higher temperatures, but the effect was not nearly so marked as in the case of metal wires. In fact, in some instances the rigidity apparently diminished slightly on keeping the fibre for some time at a high temperature.

In addition to the determination of the periods of vibration, observations of the logarithmic decrements of the amplitudes of the oscillations were taken at each temperature, in order to obtain the correction to apply to the periods for the damping of the torsional oscillations.

As an example of the observations taken I give below the figures obtained with the first fibre experimented on over this range of temperature. The periods in the table are each the mean of many observations. They have been corrected for clock rate, for the expansion of the fibre and vibrator, and also for the damping of the vibrations.

Date.	Temperature.	Corrected period in seconds.	Modulus of rigidity in 10^{11} dynes per sq. centim.	Logarithmic decrement of oscillations.	Number of hours during which temperature had been maintained.
March 25. .	14·87	8·30406	3·01754	·01234	—
„ 26. .	14·67	8·30400	3·01759	·01245	—
„ 26. .	33·14	8·29442	3·02456	·01298	2·75
„ 26. .	33·22	8·29445	3·02454	·01310	5·00
„ 26. .	33·45	8·29457	3·02445	·01306	7·75
„ 27. .	14·88	8·30367	3·01783	·01245	—
„ 27. .	54·44	8·28435	3·03192	·01383	3·75
„ 27. .	55·02	8·28417	3·03205	·01382	6·00
„ 27. .	55·20	8·28365	3·03243	·01388	8·30
„ 28. .	15·46	8·30284	3·01843	·01258	—
„ 28. .	72·47	8·27559	3·03834	·01422	3·55
„ 28. .	72·67	8·27569	3·03827	·01433	6·20
„ 28. .	72·78	8·27544	3·03845	·01433	8·75
„ 29. .	16·30	8·30234	3·01879	·01260	—
„ 29. .	97·38	8·26400	3·04687	·01511	3·10
„ 29. .	97·39	8·26417	3·04674	·01514	5·55
„ 29. .	97·41	8·26412	3·04678	·01523	8·00
„ 30. .	14·80	8·30298	3·01833	·01253	—
„ 30 .	97·06	8·26436	3·04660	·01515	2·50
„ 30. .	97·13	8·26420	3·04672	·01526	4·75
„ 31. .	15·37	8·30287	3·01841	·01256	—
April 20. .	15·90	8·30242	3·01874	·01245	—
„ 20. .	97·67	8·26362	3·04715	·01509	3·00
„ 20. .	97·69	8·26398	3·04689	—	5·55
„ 20. .	97·77	8·26415	3·04676	·01502	8·00
„ 21. .	14·98	8·30281	3·01845	·01238	—

From the numbers given in the table it will be seen that, after the first heating of the fibre, the rigidity at the lower temperature does not come back to its original value, but is slightly greater than it was before heating. After the heating to 55° C. the modulus at the ordinary laboratory temperature seems to have attained a constant value; for the values obtained after this, on being corrected for the small differences of temperature, are all in very good agreement.

In Diagram I. the mean value of the modulus of rigidity at each temperature is

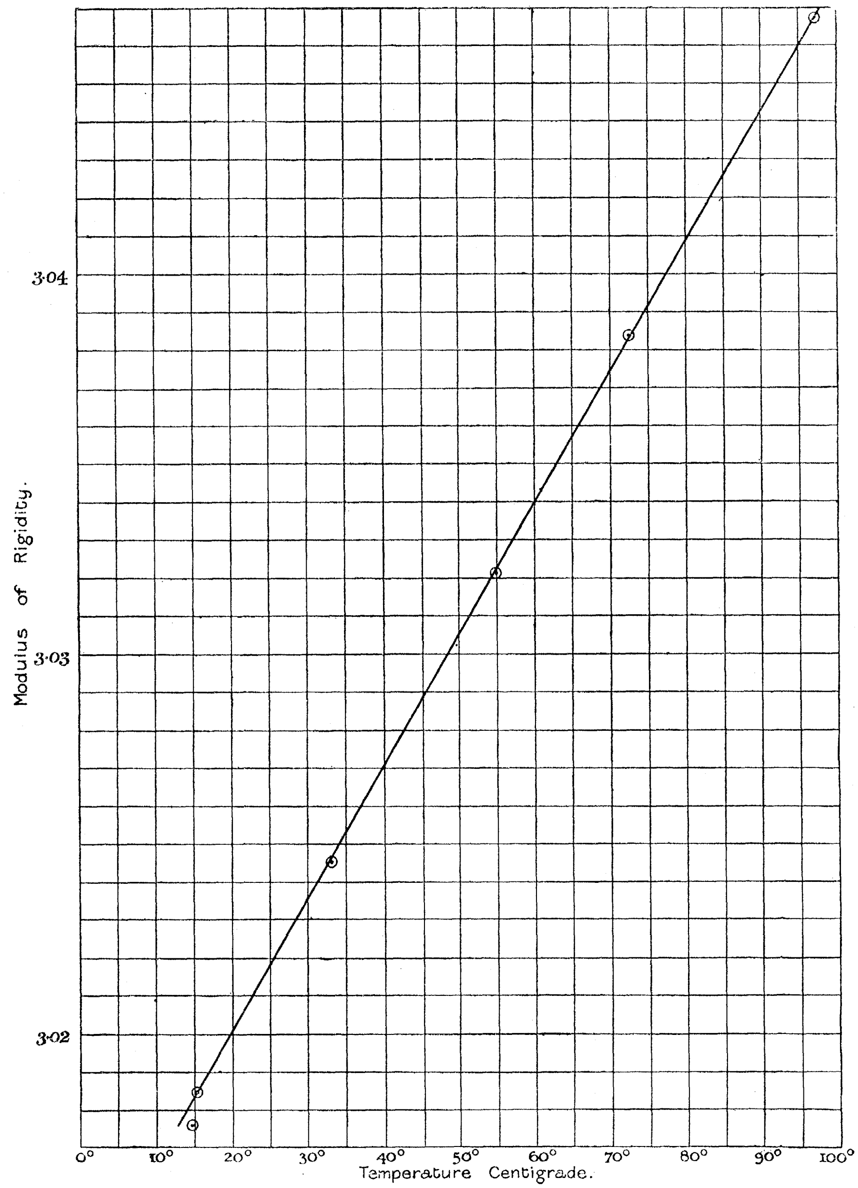

Diagram I. Showing alteration of modulus of rigidity with change of temperature.

plotted against the corresponding temperature. It will be seen that, with the exception of the mean of the first three observations at the ordinary temperature of the room, the points lie on a straight line.

The first few heatings probably have an annealing effect on the fibre, and assist it

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in accommodating itself to a definite elastic state. It is only after the fibre has been so treated that its modulus of rigidity assumes a steady value. This, over the range of temperatures investigated, is a linear function of the temperature.

It will be seen from the table that the effect of continued heating was sometimes to increase the rigidity, and sometimes to diminish it. The alteration as time went on was in all cases small, and the values of the modulus obtained by repeated heating to any one temperature were much more constant than in the case of any of the metal wires experimented on in a former research, with the exceptions of copper and of steel. It will be observed that after the fibre had been allowed to rest for some three weeks (from March 31st to April 20th) its rigidity had increased slightly, but that on heating to 100° C. it re-accommodated itself back to the normal value.

From the line drawn in Diagram I. the value of the modulus at 0° C. was found by exterpolation, and the temperature coefficient α was determined. The value found was $\alpha = \cdot 0001146$, where $n_t = n_0(1+\alpha t)$, n_0 and n_t being the values of the modulus at 0° C. and t° C. respectively. This fibre was No. 4 in the list of absolute values already given. The value given in the list as the modulus of rigidity of this fibre at 15° C. was determined immediately after the fibre had been brought to the state of constant rigidity.

Similar experiments were performed with three other fibres, and the general behaviour of each was identical with that described above. The values of the temperature coefficient obtained from these fibres were, however, considerably different, as will be seen from the following table :—

Fibre.	Mean radius in centimetres.	Length in centimetres.	Temperature coefficient of the modulus of rigidity.
4	·0011239	14·049	·0001146
5	·0007078	14·338	·0001152
6	·0005128	13·6055	·0001409
7	·00048 (about)	10·646	·0001232

Fibre 7 was unfortunately lost before its radius had been determined. The value given above was calculated by assuming the value of the modulus of rigidity of the fibre. The mean value of the temperature coefficient is $\alpha = \cdot 0001235$, but the results of these experiments show that this is far from being a constant value, and that its variation in different fibres is greater than that of the modulus of rigidity itself.

With fibres 4 and 6, at the end of the rigidity determinations, a series of observations was taken to ascertain the manner in which the logarithmic decrement and the torsional period varied with the amplitude of vibration, amplitudes up to about 10° being used. It was found that both the logarithmic decrement and the torsional period remained constant within these limits. In this respect quartz differs from metal wires, in which it was found that both the internal friction and the period of torsional vibration increased with the amplitude of oscillation.

The damping of the torsional oscillations observed in these experiments is, of course, partly due to loss of energy from the vibrating system to the surrounding air, and partly to the internal friction of the suspending fibre. In order, therefore, to obtain a measure of the viscosity of quartz, it is necessary to find what proportion of the observed logarithmic decrement is due to the friction of the air on the vibrating system. For this purpose a fibre was suspended in the jacket and the logarithmic decrement of the amplitudes of its torsional oscillations was observed. The apparatus (which had previously been made air-tight by using an indiarubber bung instead of the adjustable metal-head D, fig. 2) was then filled with hydrogen and the logarithmic decrement again observed. The following table contains the results of experiments with three fibres at 15° C. :—

Approximate length in centimetres.	Approximate radius in centimetres.	Approximate period in seconds.	Logarithmic decrement in air.	Logarithmic decrement in hydrogen.	Logarithmic decrement due to quartz fibre.	Percentage of the logarithmic decrement in air which is due to fibre.
11·45	·000638	23·30	·03392	·01714	·00111	3·27
11·45	·000846	13·27	·01964	·00984	·00048	2·44
7·40	·00112	6·00	·00966	·00484	·00024	2·49

The numbers in the column headed "Logarithmic decrement due to quartz fibre" were calculated from the observed logarithmic decrements in air and in hydrogen, using the values of the coefficient of viscosity of these gases given by O. E. MEYER.* The vibrator used in these experiments was the one used in the rigidity experiments, and the above results show that, when this vibrator is used at 15° C., only about 2·5 per cent. of the observed damping is due to the viscosity of the suspending fibre, and that this proportion is roughly constant for different fibres.

From the table already given with the observed logarithmic decrements at different temperatures it is seen that at 14·87° C. the logarithmic decrement was ·01234. Taking 2·5 per cent. of this as being due to the viscosity of the quartz fibre, the logarithmic decrement due to air-damping is ·01203. The increase in this with rise of temperature was calculated by means of the results of HOLMAN'S research on the variation of the viscosity of air with temperature.† In the following table the mean values of the observed logarithmic decrements at the different temperatures are collected together. The value of the logarithmic decrement due to air-damping at each temperature is given, and the difference between this and the observed

* O. E. MEYER, 'Kinetic Theory of Gases,' p. 171.

† S. W. HOLMAN, 'Phil. Mag.,' V., vol. 21, p. 199, 1886.

logarithmic decrement gives the logarithmic decrement due to the viscosity of the quartz fibre :—

Temperature.	Observed logarithmic decrement.	Logarithmic decrement due to air-damping.	Logarithmic decrement due to viscosity of fibre.
15·25	·01248	·01204	·00044
33·27	·01305	·01261	·00044
54·89	·01384	·01329	·00055
72·64	·01429	·01384	·00045
97·40	·01514	·01461	·00053

It thus appears that the internal viscosity of a quartz fibre varies in an irregular manner with increase of temperature. It must, however, be noticed that the internal viscosity plays but a small part in the damping of the torsional oscillations, and that a small error in the observed logarithmic decrement would make a large difference in the value obtained for the logarithmic decrement due to the viscosity of the fibre. These numbers can only be taken as showing that the viscosity of a quartz fibre is very small, and that it does not increase to any great extent when the temperature of the fibre is raised from 15° C. to 100° C.

Part III.—The Variation of the Modulus of Rigidity between 20° C. and 1000° C.

In these experiments the apparatus represented in fig. 5 was used. The fibre A was much thicker than those used in the previous experiments, and was drawn down by hand from the two quartz rods B and C. These are about 2 millims. in diameter and 8 centims. in length. The upper end of B was silvered, coppered, and then soldered into a hole along the axis of the brass rod E, from which the vibrating system is suspended. E can be clamped at the desired level in the torsion head by means of the milled-headed screw H. The lower end of C was similarly soldered into the brass rod D, which, with the cylinder F, forms the vibrator. On the rod D a small plane mirror M is fixed, and this serves, as in the former experiments, for observing the torsional oscillations. The fibre, quartz rods, and part of the brass rods E and D are surrounded by a platinum tube 7 millims. in diameter and 35 centims. in length, which was kindly lent to me for this experiment by Dr. H. A. Wilson, of Trinity College. The centre portion of this tube is heated by means of an alternating current from a transformer, and its temperature is obtained by means of a thermocouple of wires of platinum and an alloy of platinum with 10 per cent. of rhodium, welded on to opposite sides of the platinum tube at its middle, as shown in fig. 5. These wires were about ·04 millim. in diameter, and their other ends, which were some 30 centims. away, were soldered on to the wires from the galvanometer, the junctions being enclosed in a water-jacket, through which a stream of water at a known constant temperature was kept circulating.

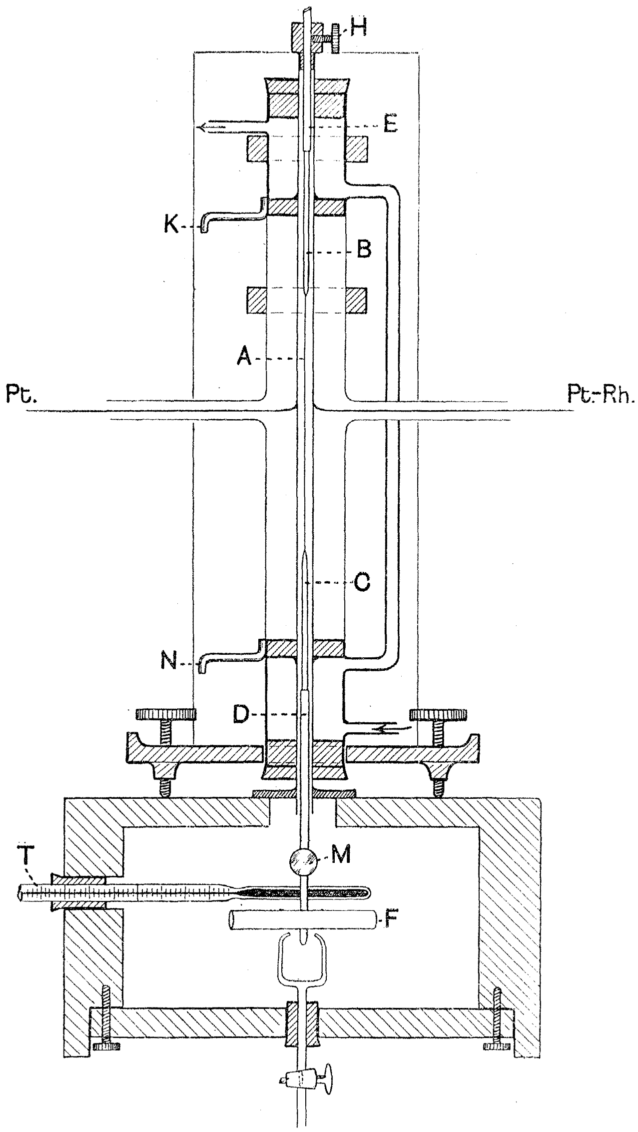

Fig. 5.

In order that the soldered junctions of the quartz rods B and C with the metal rods D and E should not get heated, two small water-jackets were fitted near the ends of the platinum tube. These surround the tube at the parts containing the soldered joints and effectively prevent its temperature from rising. The water-jackets are two brass tubes, 2·5 centims. in diameter and about 5 centims. in length. The lower end of the top jacket and the upper end of the bottom jacket are made of copper, 5 millims. thick, and these fit tightly round the platinum tube, the junction being made watertight by means of a thin layer of CHATTERTON'S cement on the

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inside. K and N are two copper rods which serve as terminals for connecting the platinum tube to the secondary of the transformer. They were bent, as shown in the figure, and dip into two mercury cups. The other ends of the two water-jackets are closed by indiarubber bungs, and the water circulates in the manner indicated by the arrows in the figure.

The part of the platinum tube to be heated is surrounded by a glass jacket of the same diameter as the water jackets, two side tubes being provided for the wires of the thermojunction. This jacket serves to keep off draughts. It is covered externally with layers of sheet asbestos, which also assist in maintaining the part of the tube surrounding the fibre at a uniform temperature.

The platinum tube, with its accessories, is supported on a stand which rests on levelling screws above a stout wooden box, rigidly fixed to the wall of the laboratory. The vibrator hangs inside this box, as shown in the diagram. The dimensions of the box are 18 centims. square by 10 centims. deep, and there is a hole in the centre of the top through which the end of the platinum tube protrudes. In order to make an air-tight joint where the tube enters the box, and yet to place no restriction on its expansion when heated, the hole in the box was covered with a piece of sheet indiarubber, through the centre of which a small hole had been made, and this fits tightly round the platinum tube. The front and bottom of the box can be removed, and the suspended system can be lowered by means of the rod E so as to fix the vibrator in position. There is a small window of optically worked glass in the front of the box, and through this the mirror M is viewed.

The only manner in which the platinum tube is held is by a clamp round the top water-jacket; it is thus free to expand in a downward direction, and no bending occurs when it is heated. The glass jacket is held in position by a second clamp, shown in the figure.

The vibrator is set in oscillation by the usual device, already described, and its temperature is indicated by the thermometer T, which enters through the side of the wooden box and has its bulb about a millimetre distant from the cylinder.

Before the experiments with the quartz fibres were begun, some experiments were made to ascertain how nearly the temperature as indicated by the thermocouple could be taken to be that of the inside of the tube; and also how the temperature inside the tube varied over the 10 centims. occupied by the fibre. For this purpose a second junction of the same two metals, viz., pure platinum and rhodo-platinum, was made and arranged on a mica frame, so that it could be placed at any desired position inside the platinum tube. The galvanometer used was a sensitive one of the d'Arsonval type. This was connected in circuit with a resistance of 1000 ohms and a key by means of which either junction could be included in the circuit. With this arrangement it was found that the temperature inside was never very different from that of the outside, the greatest difference obtained being 20° C. when the platinum tube was at 1000° C. The temperature was also constant to within these limits over the part of the tube surrounding the fibre. It was found to be impracticable to standardise the outer junction by comparison with the inner one, because the difference in their temperatures, although small, varied from time to time, the variation being probably caused by the cooling effects of air currents.

At the end of the experiments with the quartz fibres the thermocouple was standardised. For this purpose use was made of the determinations of the melting-points of potassium and sodium sulphates, and of the boiling-point of sulphur given by HEYCOCK and NEVILLE.* The platinum tube was supported horizontally, and some very small grains of potassium sulphate were placed upon it in the neighbourhood of the thermocouple. These were viewed by means of a microscope, and the temperature of the tube was gradually increased. The galvanometer deflection corresponding to the melting-point of the salt (1066° C.) was thus determined with accuracy, repeated observations giving identical results. The temperature of the other junctions (of the thermocouple wires with the galvanometer leads) was constant to within ·4° C. throughout the whole series of observations, and consequently no correction to the reading had to be made on this account.

Similar experiments were made with sodium sulphate, and the deflection corresponding to 883° C. was obtained.

In obtaining the deflection corresponding to the boiling-point of sulphur, the junction was removed from the platinum tube and placed inside a thin glass tube, which was then inserted into a retort from which sulphur was distilled. The junction was placed above the boiling liquid and well within the vapour. The barometer was read, and the boiling-point of sulphur calculated from the formula given by HEYCOCK and NEVILLE. The galvanometer deflection corresponding to the boiling-point of water was also determined in a manner similar to that employed with sulphur.

The galvanometer deflections obtained from these experiments were then plotted against the corresponding temperatures, and from the resulting curve the temperature corresponding to any deflection could be read.

Three fibres were experimented on over this range of temperature. They were each about 10 centims. in length, and about ·006 centim. in diameter. These were made by drawing down the centre of a quartz rod fairly thin, and then heating the thin part and drawing it out suddenly. Practice was required before fibres of the right length and sufficiently thin could be made by this method, but after a few attempts no difficulty was experienced.

Observations of the period of torsional vibration and of the logarithmic decrement of the amplitudes were taken at intervals of about 50° C. as the temperature was increased from that of the room to 1000° C., at which temperature the internal friction of the fibres was so great that the vibrations were nearly dead-beat. The fibres experimented on were made from different specimens of quartz, but the results obtained were in every case the same. The rigidity of the fibre increased with the

* 'Jour. Chem. Soc.,' vol. 67, p. 160.

temperature, at first as a linear function of it, but as the temperature rose the rate of increase gradually diminished, and a maximum rigidity was attained at about 880° C. On passing this temperature, the rigidity decreased very rapidly with increase of temperature.

The logarithmic decrement of the torsional oscillations increased with the temperature at a rate which was at first constant, but after about 650° C. it became much more rapid. In order to see what part of the observed logarithmic decrement was due to the viscosity of the fibre, several fibres of about the same dimensions as those used in these experiments, and carrying the same vibrator, were in turn suspended in a large air-tight brass tube and made to vibrate first in air and then in hydrogen, the logarithmic decrements of the torsional oscillations being observed in each gas. The values of the logarithmic decrement due to the fibre were then obtained in the same way as with the finer fibres. The values given differed very considerably even when obtained from repeated observations with the same fibre. This is no doubt largely due to the smallness of the observed logarithmic decrements. The results showed that from 0 to 5 per cent. of the observed logarithmic decrement in air at 17° C. was due to the viscosity of the quartz fibre. In correcting the observed logarithmic decrements in the experiment proper, 2·5 per cent. of the observed logarithmic decrement at 20° C. was taken as being due to the viscosity of the fibre, the remaining 97·5 per cent. being due to air-damping. This is, of course, only an approximate value, but is, I think, good enough for the present purposes, for the internal viscosity of quartz at the ordinary temperature of the laboratory is extremely small in comparison with the value it has at high temperatures, and, consequently, a difference of 50, or even 100, per cent. in the value at 20° C. would be hardly noticeable in plotting a curve to represent the changes up to 1000° C. The temperature of the air surrounding the vibrator was constant during the experiments, and the logarithmic decrement due to air-damping was taken to have the same value throughout the experiments. This would not be exactly the case, for the viscosity of the air surrounding the part of the vibrating system the temperature of which is raised would increase with the temperature. The effect of this increase would, however, probably be small in comparison with the whole damping effect due to the air, for most of the damping of the torsional vibrations would doubtless be caused by the metal cylinder (F, fig. 5), the temperature of which remains constant. It will also be seen from the appended table that the logarithmic decrement due to air-damping is itself a small fraction of the actual observed logarithmic decrement at high temperatures.

Temperature Centigrade.	Corrected period of the torsional oscillations in seconds.	Modulus of rigidity in 10^{11} dynes per sq. centim.	Observed logarithmic decrement of the amplitudes.	Logarithmic decrement due to the internal viscosity of the quartz fibre.
20·52	3·88202	3·0017	·001789	·000044
170	3·85441	3·0448	·001812	·000067
296	3·83197	3·0805	·001823	·000078
404	3·81632	3·1059	·001913	·000168
483	3·80646	3·1219	·002008	·000263
613	3·78929	3·1504	·002440	·000695
656	3·78497	3·1575	·002846	·001101
673	3·78292	3·1610	—	—
700	3·77994	3·1660	·004043	·002298
766	3·77490	3·1744	·007781	·006036
795	3·77310	3·1775	·01041	·00867
833	3·77313	3·1775	·01443	·01269
876	3·77163	3·1799	·01967	·01793
919	3·77168	3·1798	·02447	·02273
948	3·77416	3·1757	·03009	·02835
988	3·78588	3·1560	·04165	·03991
1005	3·79612	3·1391	·07547	·07373
1008	3·8337	3·0778	·1662	·1645
1008	3·8497	3·0523	—	—
1058	over 4	less than 2·83	·335	·333
22·10	3·89305	2·9847	·001762	·000017

The values of the modulus of rigidity contained in the above table were calculated from the observed periods of torsional oscillation, on the assumption that the modulus of rigidity of the fibre at 15° C. was $3{\cdot}001 \times 10^{11}$ dynes per sq. centim.—the mean value found in the first part of this research. In Diagram II. these values, and also the logarithmic decrements due to the internal viscosity of the quartz fibre, are plotted against the corresponding temperatures, and from the curves drawn the effects of increased temperature can be readily followed. That quartz should have a temperature of maximum rigidity was, of course, to be expected, seeing that at ordinary laboratory temperatures its rigidity increases with rise of temperature, and that it must become zero as the quartz melts. From Diagram II. it will be seen that this temperature of maximum rigidity is about 880° C.

In the experiments in the range up to 100° C. it was observed that the rigidity at the ordinary temperature of the laboratory was slightly increased by heating the fibre. This was also found to be the case in the experiments just described, so long as the fibre was not heated above 760° C. The rigidity at the laboratory temperature was always diminished by heating the fibre to higher temperatures than this.

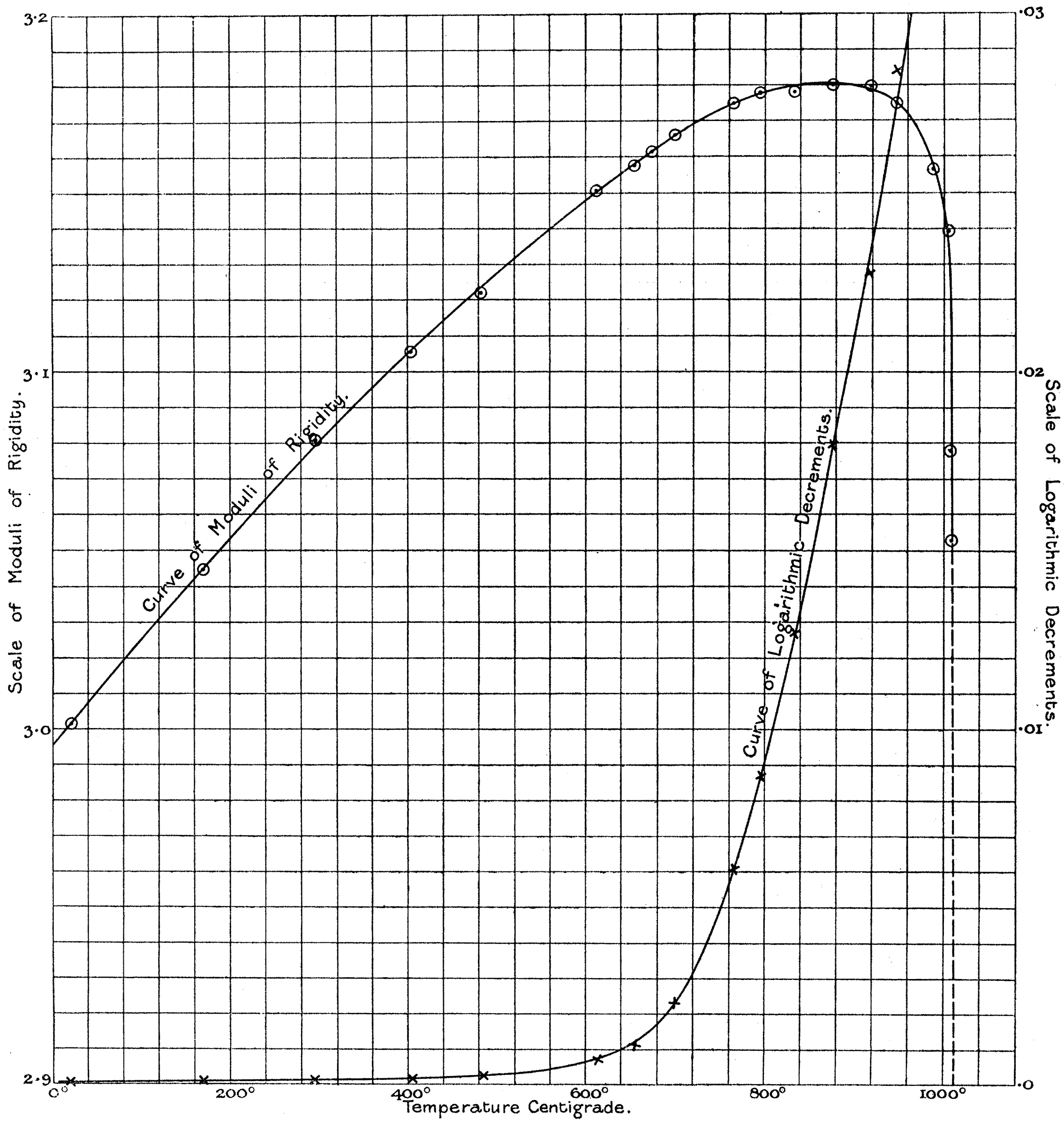

Diagram II. Showing effect of change of temperature on modulus of rigidity and on logarithmic decrement of torsional vibrations.

Comparison of Results and Conclusion.

The mean values of the modulus of torsional rigidity (n) and its temperature coefficient between 15° C. and 100° C. (α), obtained by the observers mentioned at the beginning of this paper, are collected together in the following table, which also contains the values arrived at in the present research :—

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Observer.	Value of n in 10^{11} dynes per sq. centim.	Value of α.
Boys	2·38	—
Threlfall	2·8815	·000128
Barnett	1·46	·000115
Horton	3·001	·0001235

It is difficult to see why the value of the modulus of rigidity obtained by Barnett is so much lower than those found by the other observers. Barnett states that the fibres used by him were much thicker than those used by Boys or Threlfall, and that it is probable that the rigidity of fine fibres, like the breaking weight per unit area of cross-section, increases as the diameter decreases.* This, however, is not borne out by the fact that some of the fibres used by Threlfall were very nearly of the same diameter as some of those used by Barnett. Boys and Threlfall both found considerable variations between the values of the rigidity modulus obtained from different fibres. These observers used a high-power microscope to measure the diameter of the fibres directly, and Threlfall states that "it is impossible to be sure of the thickness of threads of about ·015 centim. diameter within less than 3 or 4 per cent. by a single measurement." It is therefore probable that the discrepancies found were largely due to an inaccurate knowledge of the radius of the fibres used; indeed, it must have required great skill in experimenting, and patience in taking many observations of the diameters, in order to obtain as good an agreement as that given.

Of the values of the temperature coefficient of the modulus between 15° C. and 100° C., that given by Threlfall was obtained from a single fibre. The value given by Barnett was the mean of the results of observations on several fibres. The variation between the different values was considerable, but not so large as the variation found in the present experiments. The fibres used by Barnett were, however, all prepared from one crystal of quartz, and the results obtained would therefore be expected to be more uniform.

Barnett also made observations to determine if there was any change of rigidity with time. Experiments extending over an interval of four months failed to detect any alteration. The present experiments show that with more accurate timing of the torsional oscillations, variations in the rigidity can be detected, and that, when a fibre is allowed to rest at the ordinary temperature of the laboratory, its rigidity increases slightly. The increase of rigidity with time is, however, not so marked in the case of quartz as it was found to be with metal wires, and the reason for this may possibly lie in the fact that the structure of fused quartz is amorphous, while

* It is, of course, well known that the tensile strength of metal wires also increases as their diameter is decreased.

that of a metal wire is crystalline. It is known that the effect of heat is to cause the crystalline grains of which a metal is composed to grow, and this alteration of the structure would mean an alteration in the rigidity of the material. The rate of growth of the crystals at 100° C. would be very small, and it has been observed only in the case of the softer metals, but it is probable that the present method can detect changes which the microscope fails to show. If this view is correct, it would follow that the larger crystalline structure has the greater rigidity, and this is supported by some experiments on the effect of heat on the elasticity of steel recently performed in the Engineering Laboratory of this University by Mr. F. Rogers, who found that Young's modulus for steel was increased by cooking the specimen to high temperatures—a process which increases the size of the crystalline grains. On the other hand, the observed increase of rigidity with time may be merely the result of a slow annealing and due to the easing of the crystalline structure from contraction stresses. In the present case of quartz fibres such small "time effect" as exists, is, I think, undoubtedly due to a gradual annealing, and in the case of metal wires it is probably due in part to each of the causes mentioned above.

It is a curious fact that, although a quartz fibre is made quite brittle and useless for most practical purposes by being heated to about 500° C., yet its rigidity at the ordinary temperature of the laboratory is increased so long as it is not heated above 760° C.

Since the rigidity of quartz fibres is much more constant than the rigidity of any metal wires, and as also the period and the logarithmic decrement of the torsional vibrations are independent of the amplitude of oscillation so long as this is small, it appears that quartz fibres are by far the best material to use for suspensions in all forms of torsion apparatus where accuracy is required.

In conclusion, I wish to express my sincere thanks to Professors J. H. Poynting and J. J. Thomson for their kind encouragement and valuable advice throughout the course of these experiments.